DSGE Models and Calibration Exercise

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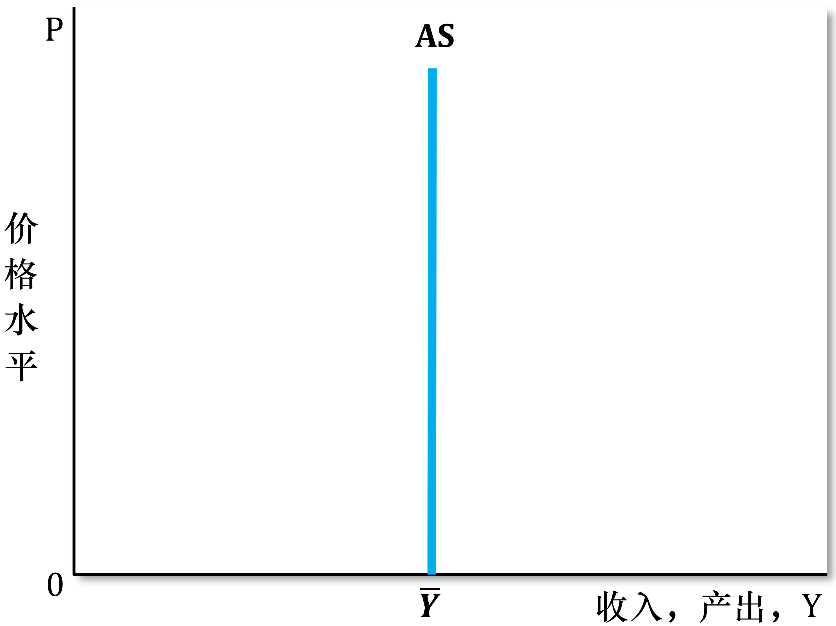
Figure 1 Macroeconomics at the Undergraduate Level: Keynesian vs. Classical Supply Functions

The classical AS curve

- ✓ The *vertical* classical supply function is based on the assumption that there is always full employment of labor, and thus that output is always at the corresponding level.



The Keynesian AS curve

- ✓ The *horizontal* Keynesian AS curve implies that any amount of output will be supplied at the existing price level.
- ✓ Price and wage rigidity in the short run.

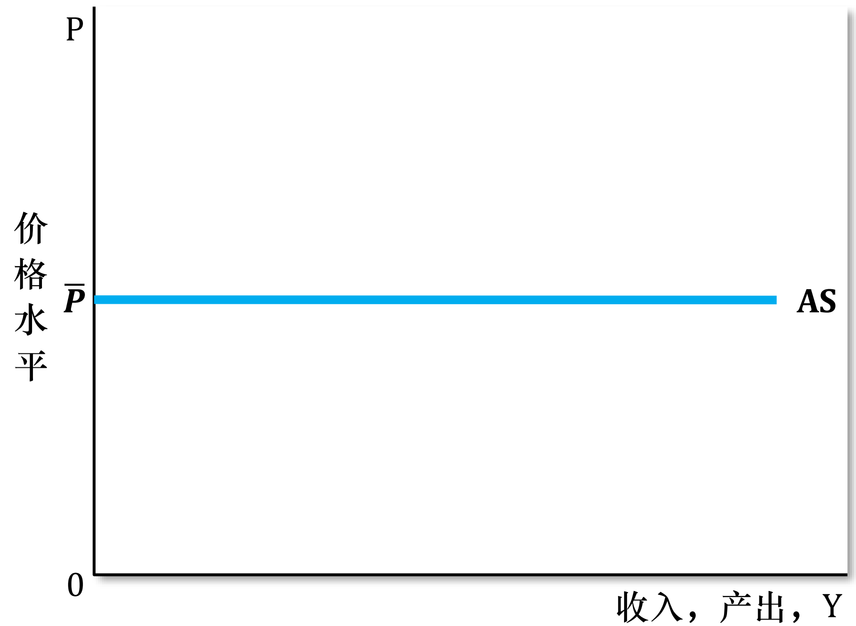
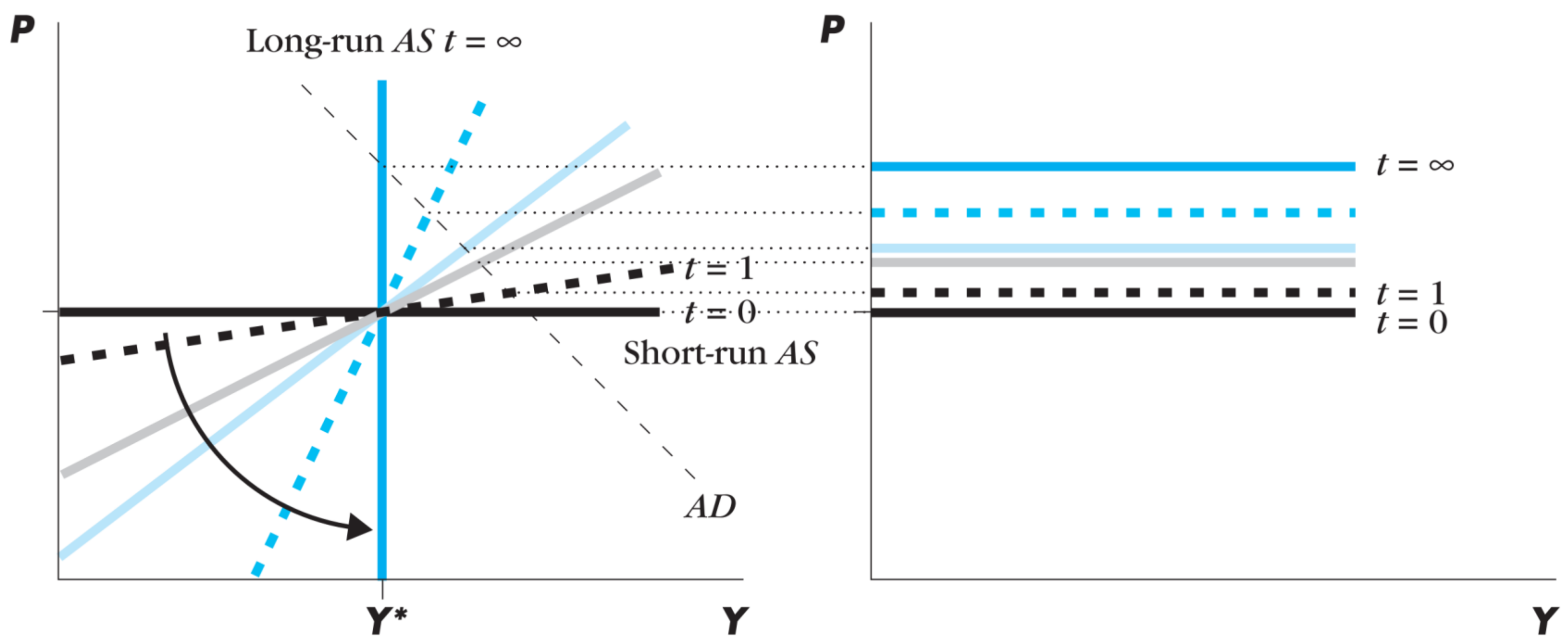


Figure 2 Macroeconomics at the Undergraduate Level: Keynesian vs. Classical Supply Functions

- The question, *How steep is the aggregate supply curve?*, is in effect the main controversy in macroeconomics.
- *In the long run*, the aggregate supply curve is *vertical*; while *in the short run*, the aggregate supply curve is *horizontal*. The aggregate supply curve describes the *price adjustment mechanism* of the economy.



Macroeconomics at the Undergraduate Level: Keynesian vs. Classical Supply Functions

- **Recall short-run aggregate supply function:**

$$\pi_t = E_t\pi_{t+1} + \lambda(Y_t - \bar{Y}_t) + \varepsilon_t$$

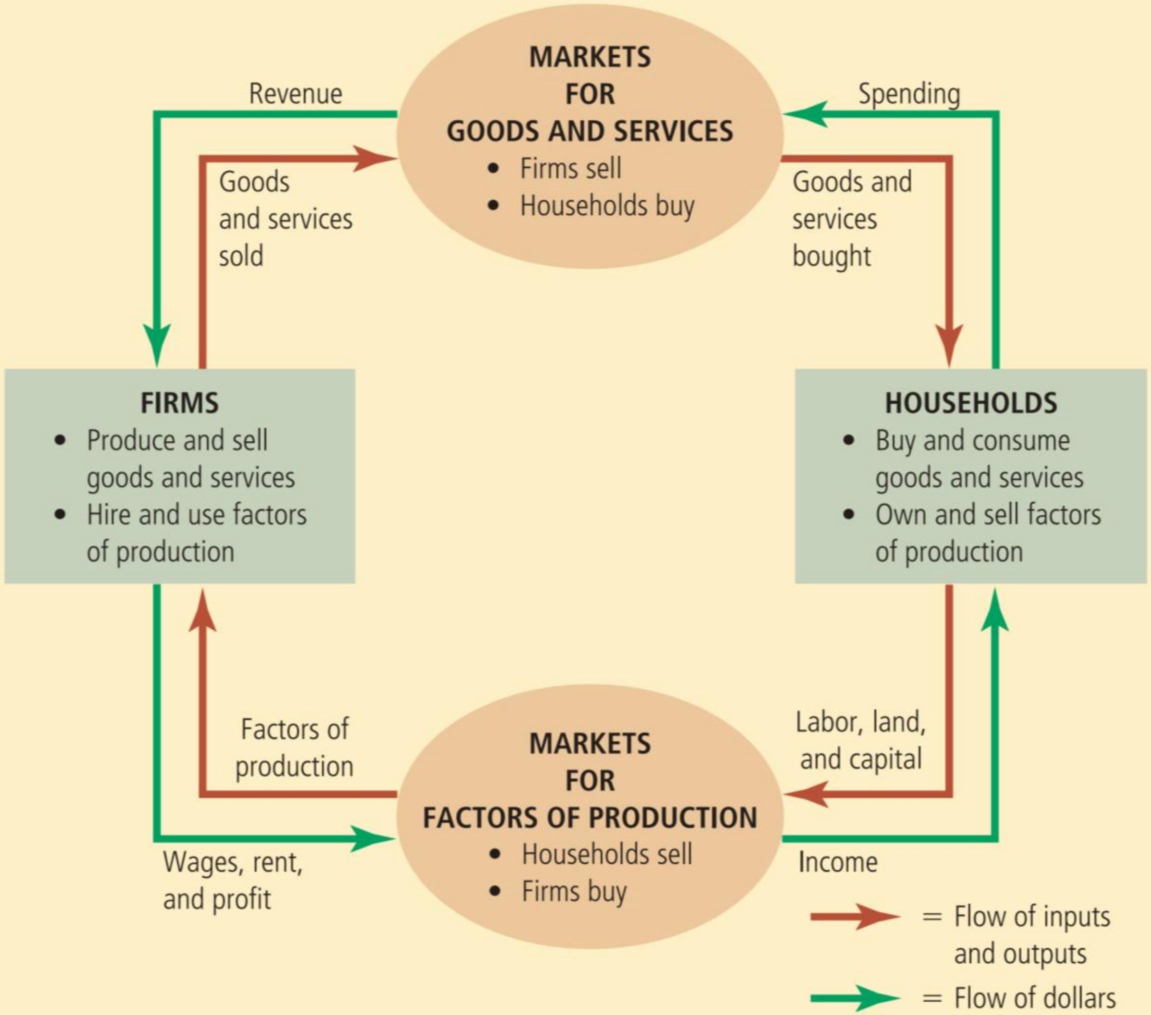
  - The speed of price adjustment is controlled by the parameter λ in the above equation.

- **How steep is the aggregate supply curve?**
  - Econometric methods
  - DSGE models
    - e.g.: aggregate supply function solved in the Calvo model

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\frac{1+\xi+\alpha(\sigma-1)}{\alpha}(Y_t - \bar{Y}_t)$$

Figure 3 Macroeconomics at the Undergraduate Level: The Circular Flow —— Two Sectors

➢ **How to depict the economic structure of interest?**



The Circular Flow
This diagram is a schematic representation of the organization of the economy. Decisions are made by households and firms. Households and firms interact in the markets for goods and services (where households are buyers and firms are sellers) and in the markets for the factors of production (where firms are buyers and households are sellers). The outer set of arrows shows the flow of dollars, and the inner set of arrows shows the corresponding flow of inputs and outputs.

Figure 4 Macroeconomics at the Undergraduate Level: The Circular Flow —— Three Sectors or More

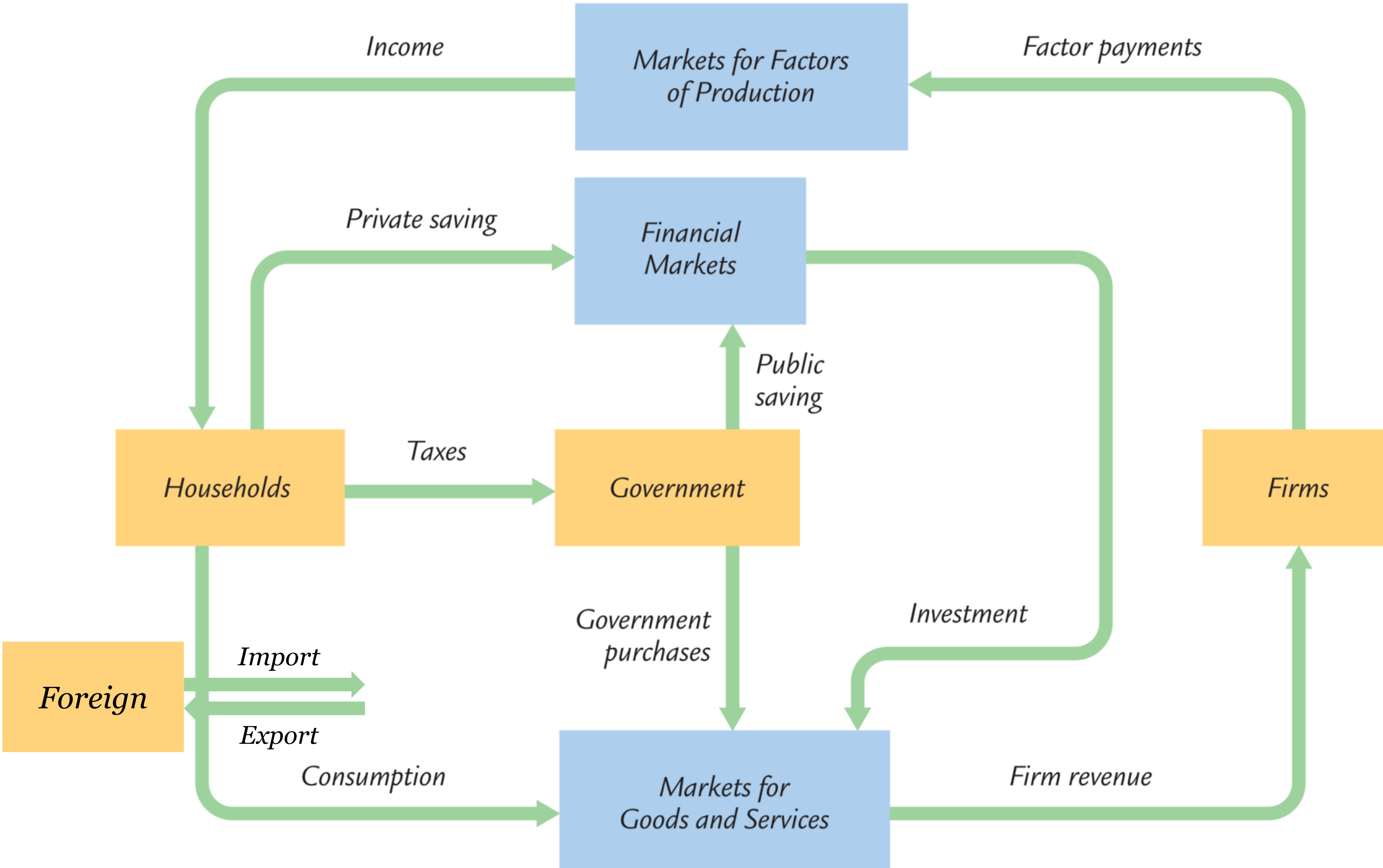


Figure 5

The Economic Structure

➤ 尝试性分析框架：基于金融开放的中国宏观经济内、外均衡

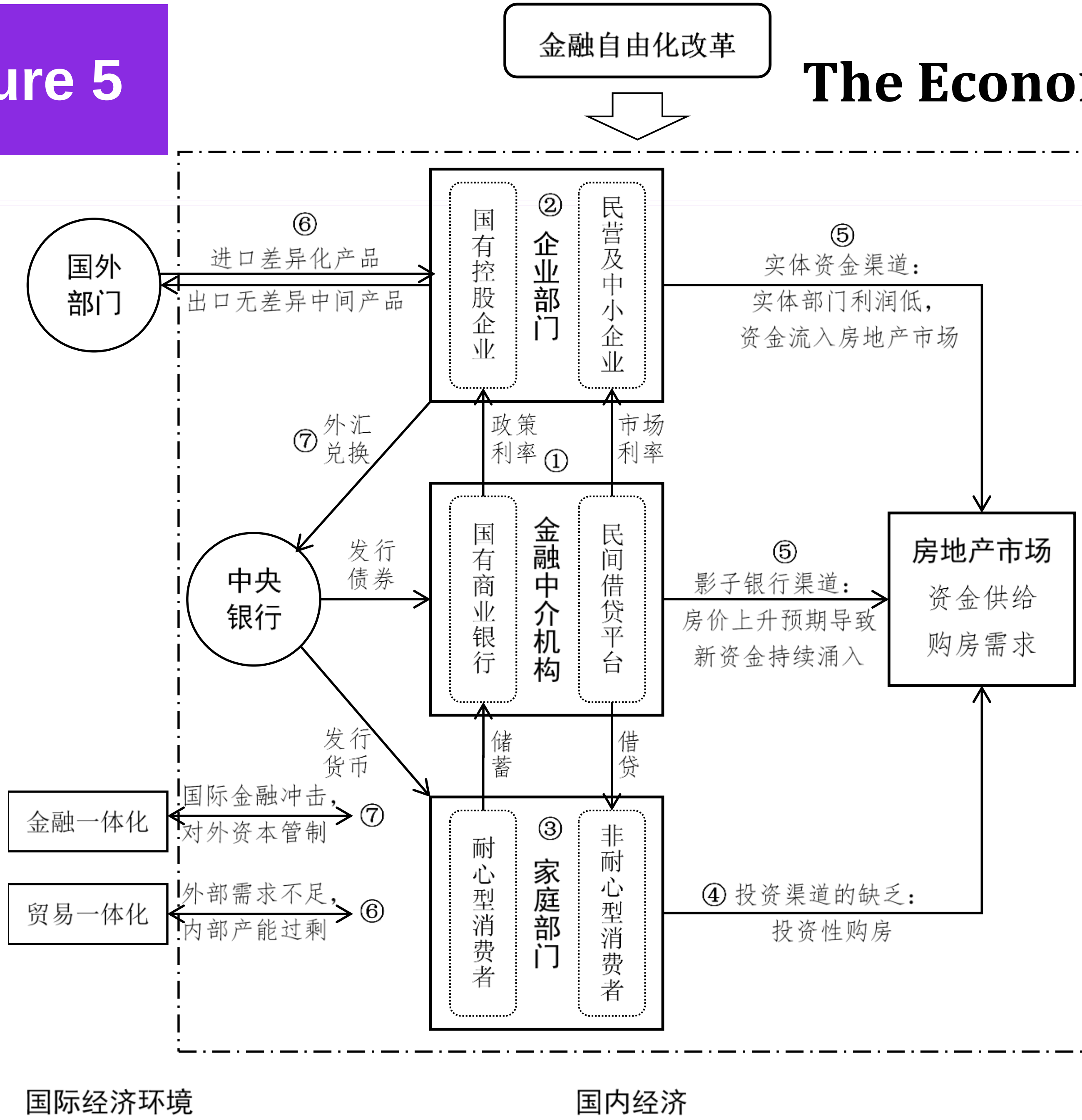
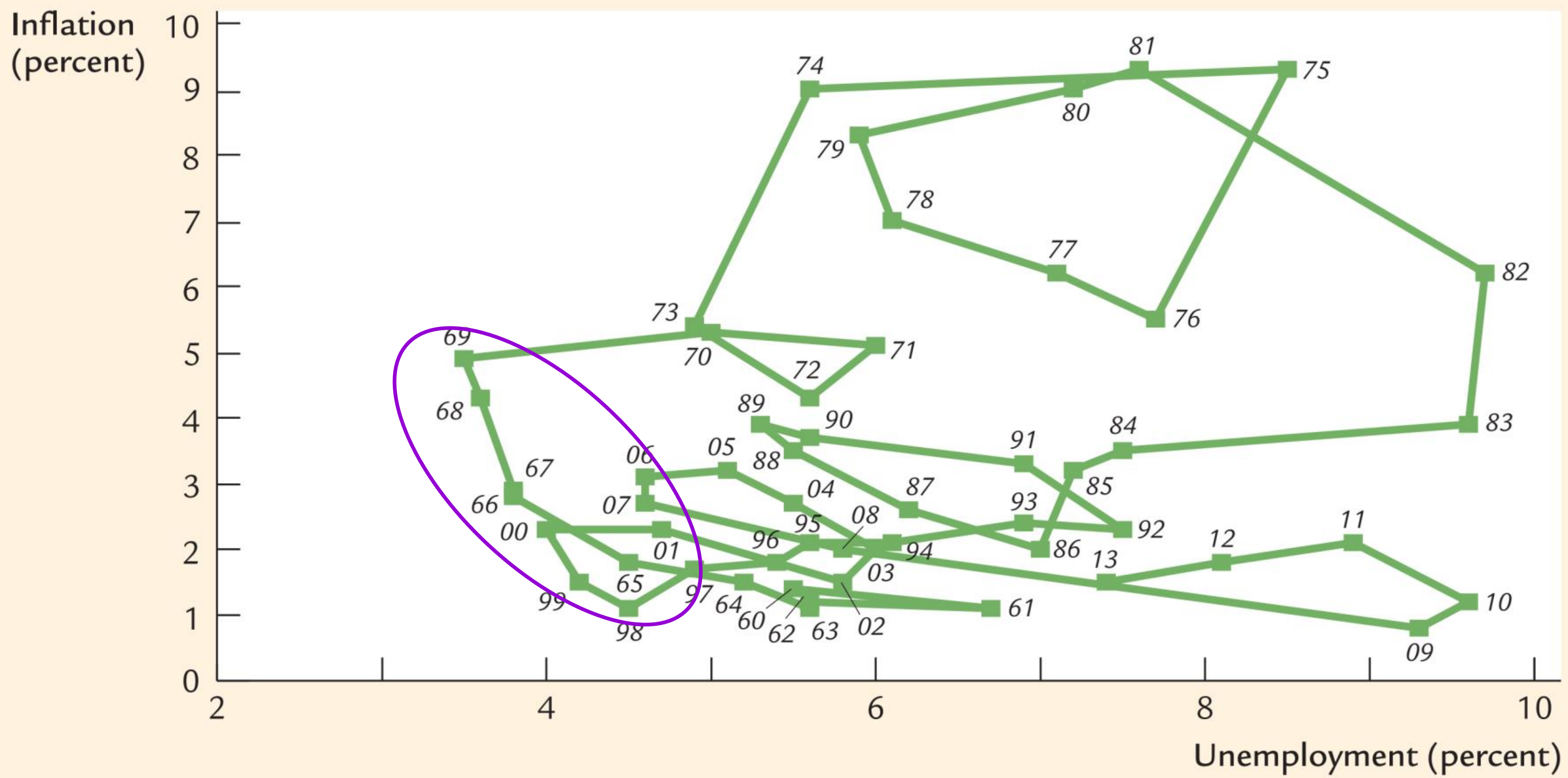


Figure 6

Toward DSGE Models
Historical Origins —— Empirical



Inflation and Unemployment in the United States, 1960–2013 This figure uses annual data on the unemployment rate and the inflation rate (percentage change in the GDP deflator) to illustrate macroeconomic developments spanning half a century of U.S. history.

Data from: U.S. Department of Commerce and U.S. Department of Labor.

Toward DSGE Models
Historical Origins —— Theoretical

- **Steven Landsburg on rational expectations revolution**

  Today there are many different models of macroeconomics. Some incorporate parts of the original Lucas story; others dispense with it completely. But they all have this much in common: They tell precise stories about individuals making decisions under changing and *uncertain conditions*, and they use substantial mathematics to keep track of *how everyone's decisions affect everyone else's*.

  ....DSGE models vary greatly in the stories they tell and the predictions they make, but they provide a common language that allows macroeconomists to pinpoint why they've reached different conclusions. ...DSGE models focus on motivations rather than just behavior.

Toward DSGE Models

- *The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them.*

—William Bragg

- **What is DSEG models?**
  - *Dynamic*: trace the path of variables over time.
  - *Stochastic*: incorporate the inherent randomness of economic life.
  - *General equilibrium*: take into account the fact that everything depends on everything else.

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Steps in an Calibration Exercise

- **Definition of calibration**

- Calibration as a collection of procedures designed to provide an answer to economic questions by using a model that approximates the data generating process (DGP) of the observable data.

- **Kydland and Prescott (1996) outlined a five-step procedure for implementing calibration exercises in application to DSGE models:**

  1. Pose a question
  2. Use well-tested theory to address the question
  3. Construct a model economy
  4. Calibrate the model economy
  5. Run the experiment

Steps in an Calibration Exercise

1. **Pose a question**
   - Questions may involve assessments of the theoretical implications of changes in policy.
   - Or they may involve assessments of the ability a model to mimic features of the actual economy.
2. **Use well-tested theory to address the question**
   - A researcher needs a theory that has been tested through use and found to provide reliable answers to a class of questions.
3. **Construct a model economy**
   - Choose functional forms for the primitives of the model and find a solution for the endogenous variables in terms of the exogenous ones and the parameters.

Steps in an Calibration Exercise

4. Calibrate the model economy

- Calibration is not estimation, the parameter values selected are not the ones that provide the best fit in some statistical sense.
- In some cases, the presence of a particular discrepancy between the data and model economy is a test of the theory been used.

5. Run the experiment

- Evaluate the quality of the model by comparing its outcomes with a set of stylized facts of the actual data.
- Propose an answer to the question, characterize the uncertainty surrounding the answer, and do policy analysis if required.

Alternative Estimation Methods

$$\textbf{\textit{Empirical Methods}}\begin{cases} \textbf{\textit{Calibration}} \\ \color{purple}{model \xrightarrow{approximate} DGP} \\ \textbf{\textit{Estiamtion}}\begin{cases} Limited\ information \Longrightarrow \textbf{\textit{Matching moments}} \\ Full\ information\begin{cases} \textbf{\textit{MLE}} \\ \textbf{\textit{Bayesian methods}} \end{cases} \end{cases} \end{cases}$$

➢ Calibration v.s. Estimation

- **Calibration**: the specification of *priors*. It's an exercise under which a set of empirical targets is used to pin down the parameters of the model under investigation, and a second set of targets is used to judge the model's empirical performance.
- **Estimation**: parameterization is accomplished via *estimation*, and empirical performance is assessed via *hypothesis testing*.

Alternative Estimation Methods
Maximum likelihood v.s. Bayesian methods

- The distinction between these two methods is whether the data or parameters are interpreted by the researcher as *random variables*.

- **Maximum likelihood**
  - Parameters are interpreted as *fixed* (but unknown), and the data are interpreted as the realization of a *random* drawing from the corresponding likelihood function.
  - Model assessment: how plausible are the observed data, given the maintained null hypothesis?

- **Bayesian methods**
  - Parameters are interpreted as *random*, and the data are *fixed*.
  - Model assessment: how plausible is a given model relative to competing alternatives, given the observed data?
  - Somewhere between calibration (prior) and MLE (data).

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Model Setup —— Households

- Households maximize utility over consumption, c_t and leisure, $1 - l_t$, where l_t is labor input, according to the following utility function:

$$\mathbf{E}_t \sum_{t=0}^{\infty} \beta [log c_t + \psi log(1 - l_t)]$$

- Subject to the following budget constraint:

$$c_t + k_{t+1} = w_t l_t + r_t k_t + (1 - \delta) k_t$$

➢ Optimality Conditions

$$\frac{1}{c_t} = \beta \mathbf{E}_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right]$$

$$\psi \frac{c_t}{1 - l_t} = w_t$$

Model Setup —— Firms

- Firms maximize profits by the choice of labor input l_{it}, capital input k_{it}, and product price p_{it}:

$$\pi_{it} = p_{it}y_{it} - w_t l_{it} - r_t k_{it}$$

- Subject to the production function:

$$y_{it} = k_{it}^{\alpha}(e^{z_t} l_{it})^{1-\alpha}$$

- z_t captures technology which evolves according to:

$$z_t = \rho z_{t-1} + e_t$$

  - where ρ is a parameter capturing the persistence of technological progress and $e_t \sim N(0, \sigma)$.

Model Setup —— Firms

➢ Optimality Conditions

- Optimal capital to labor ratio: $k_{it} r_t = \frac{\alpha}{1-\alpha} w_t l_{it}$
- The solution to the pricing problem, yields the constant markup pricing condition of *monopolistic competition*:

$$p_{it} = \frac{\epsilon}{\epsilon - 1} mc_t p_t$$

- symmetric firms implies that all firms charge the same price and thus $p_{it} = p_t$; we therefore have $mc_t = \frac{\epsilon - 1}{\epsilon}$.
- We can compute marginal cost:

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{1}{A_t} w_t^{1-\alpha} r_t^{\alpha}$$

Model Setup —— Firms

- Combining this result for marginal cost, as well as its counterpart in terms of capital, with the optimal pricing condition yields the final two important equations in our model:

$$w_t = (1-\alpha)\frac{y_{it}}{l_{it}}\frac{\epsilon - 1}{\epsilon}$$

$$r_t = \alpha\frac{y_{it}}{k_{it}}\frac{\epsilon - 1}{\epsilon}$$

- we aggregate the production of each individual firm to find an aggregate production function:

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$$

Solving the Model
The Equations in the Model

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right] \tag{1}$$

$$\psi \frac{c_t}{1 - l_t} = w_t \tag{2}$$

$$y_t = c_t + i_t \tag{3}$$

$$y_t = k_t^{\alpha} (e^{z_t} l_t)^{1-\alpha} \tag{4}$$

$$w_t = (1 - \alpha) \frac{y_t}{l_t} \frac{\epsilon - 1}{\epsilon} \tag{5}$$

$$r_t = \alpha \frac{y_t}{k_t} \frac{\epsilon - 1}{\epsilon} \tag{6}$$

$$i_t = k_{t+1} - (1 - \delta) k_t \tag{7}$$

$$z_t = \rho z_{t-1} + e_t \tag{8}$$

Table 1

Solving the Model
The Variables and Parameters in the Model

Endogenous Variables		Parameters	
y_t	Output	α	capital elasticity in production function
c_t	Consumption	β	Discount factor
k_t	Capital	δ	Capital depreciation rate
i_t	Investment	ψ	Leisure preference parameter
l_t	Labor	ρ	Persistence of technological progress
w_t	Wage rate	σ	Variance of technology shock e_t
r_t	Rate of rent	ϵ	Price markup parameter
z_t	Technology		
Exogenous Variables			
e_t	Technology shock		

Parameter Calibration

- The parameters can be **calibrated** such that the *deterministic steady state* replicates certain time series averages of actual data in the country of interest.

- **The discount factor β**

- Suppose the average real return to equity, which in the model corresponds to $1 + r - \delta$, is about 4% per annual in Country A.
- According to Equation (1), at steady state:

$$\beta = 1/(1 + r - \delta) = 1/1.04^{0.25} \approx 0.99$$

- **The capital elasticity α**

- According to Equation (6), $\alpha(\epsilon - 1)/\epsilon$ represents the capital share in GDP, which matches the real data: $r\bar{K}/\bar{Y} = \alpha(\epsilon - 1)/\epsilon = 0.3$.
- If **price markup parameter** $\epsilon = 10$, we can obtain $\alpha = 1/3$.

Parameter Calibration

➢ The capital depreciation rate δ

- According to Equation (1) (6) (7), at steady state, we can obtain:

$$\begin{cases} \beta(1+r-\delta)=1 \\ r=\alpha\frac{\bar{Y}}{\bar{K}}\frac{\epsilon}{\epsilon-1} \\ \bar{I}=\delta\bar{K} \end{cases} \Rightarrow \begin{cases} \beta\left(\alpha\frac{\bar{Y}}{\bar{K}}\frac{\epsilon}{\epsilon-1}+1-\delta\right)=1 \\ \bar{I}=\delta\bar{K} \end{cases}$$

- Suppose average ratio of investment to GDP ($\bar{I}/\bar{Y}$) in Country A is 0.2.
- Insert $\beta=0.99, \alpha(\epsilon-1)/\epsilon=0.3$ as we've obtained earlier, we can solve for $\delta=0.023$, which corresponds to an annual depreciation rate of 10%.

Parameter Calibration

➢ The leisure preference parameter ψ

- Combining Equation (2) (3) (5), we can obtain steady-state relationship as followed:

$$\psi \frac{1-\bar{I}/\bar{Y}}{1-\bar{L}} = (1-\alpha)\frac{1}{\bar{L}}\frac{\epsilon-1}{\epsilon}$$

- we pin down $\bar{L} = 0.3$, which is the fraction of time endowment spent in the workplace in the steady state.
- Insert parameter values obtained earlier, we can solve for $\psi = 1.75$.

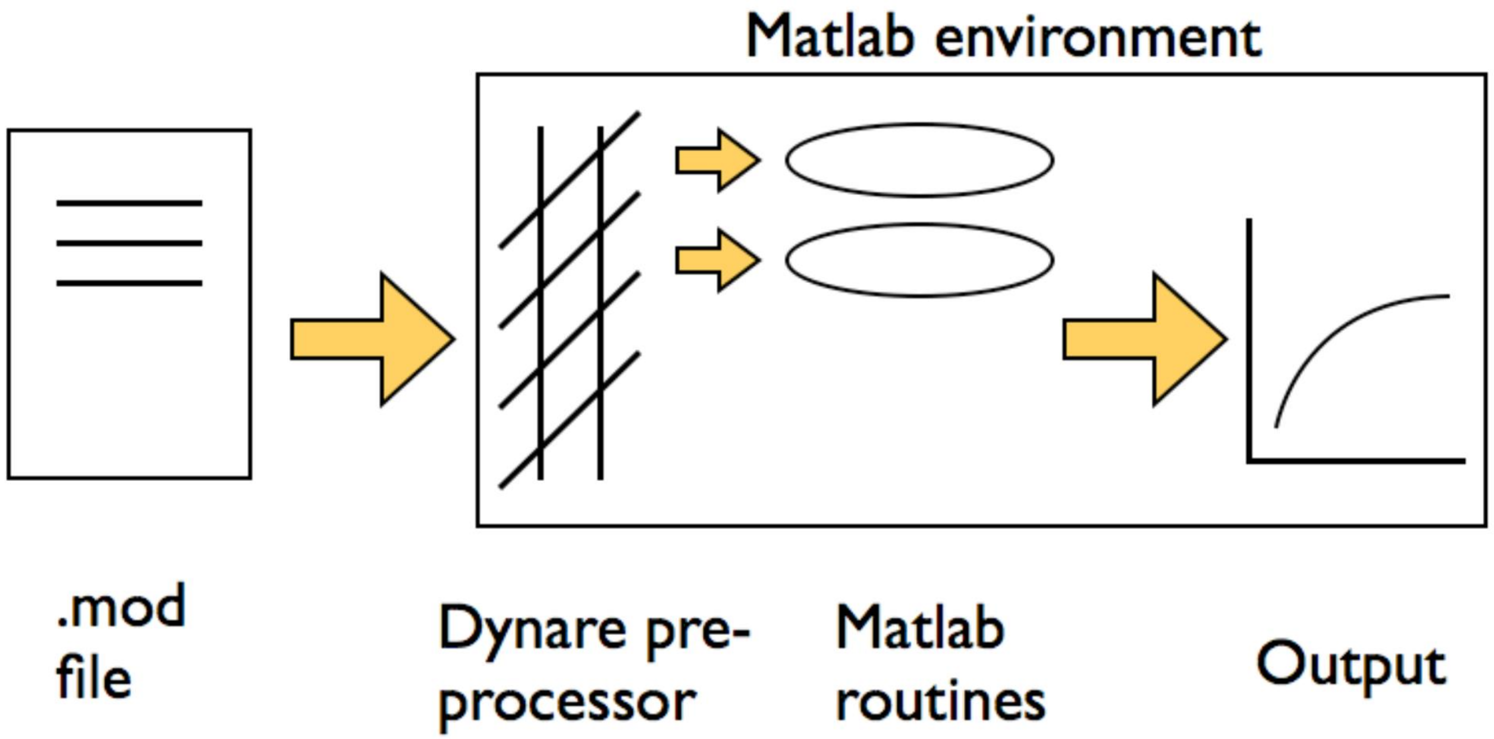
➢ The technological process ρ and σ

- We set $\rho = 0.95, \sigma^2 = 0.01^2$.

Contents

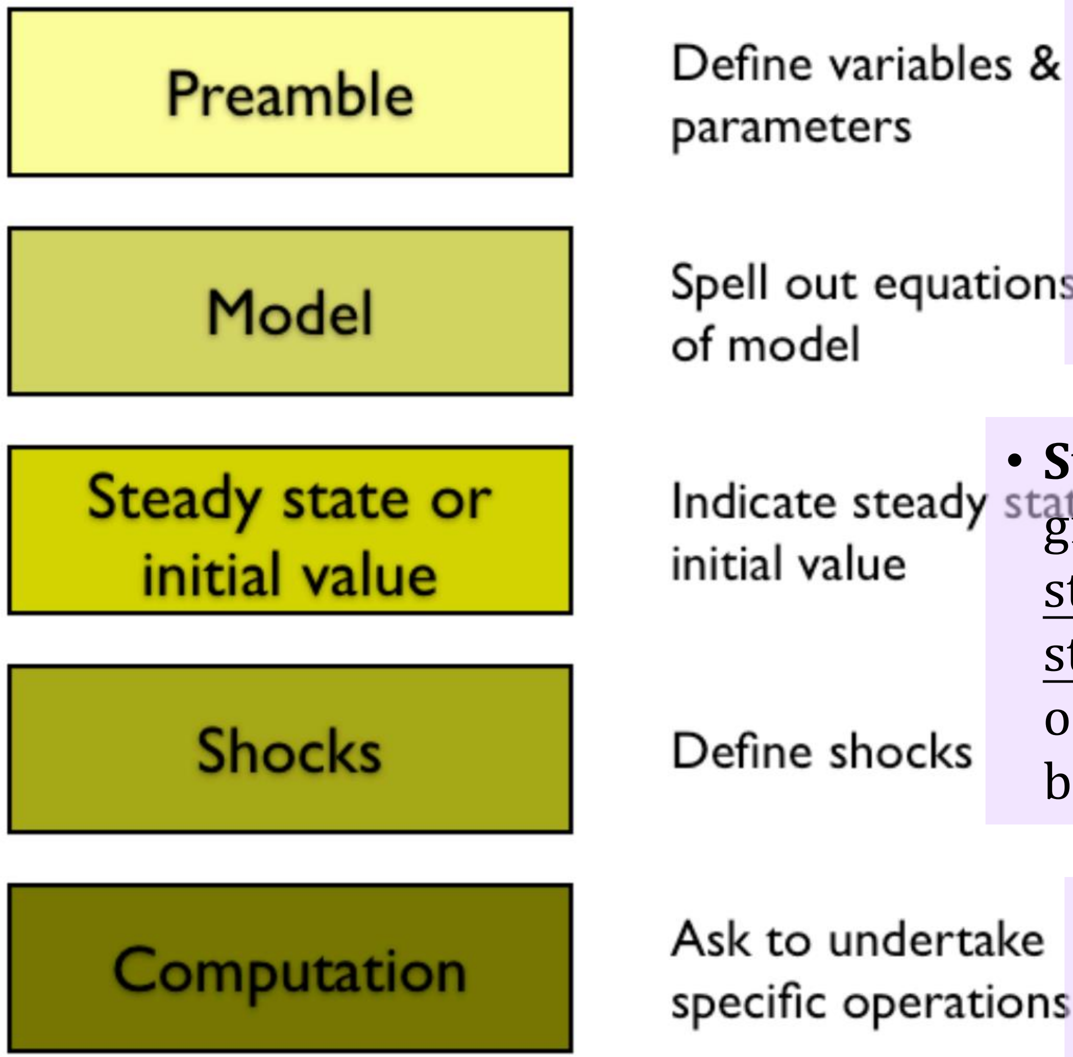
Figure 7 What is Dynare?

➢ Dynare is a powerful and highly customizable engine, with an intuitive front-end interface, to solve, simulate and estimate DSGE models.



- *The .mod file being read by the Dynare pre-processor, which then calls the relevant MATLAB routines to carry out the desired operations and display the results.*

Figure 8 Structure of the .mod file



- **Preamble:**
  Lists endogenous variables, exogenous variables that will be shocked;
  List parameters and assigns values to each.

- **Steady state or initial value:** gives indications to find the steady state of a model, or the starting point for simulations or impulse response functions based on the model's solution.

- **Computation:**
  e.g. Forecasting, estimating impulse response function.

Structure of the .mod file

- **We can write down the complete .mod files corresponding to our example in Part 3.**
- ✓ All parameters been calibrated.
- ✓ All parameters been estimated using Bayesian techniques.
    - Usually we calibrate the parameters that determine the economy's long-run characteristics.
    - And restrict the remaining parameters to intervals that are in line with the empirical evidence using estimation methods.
- **Reproduce a paper of interest**
- ✓ GarciaCicco_et_al_2010.mod
    - the mod-file was written by Johannes Pfeifer

Appendix References and Supplementary Readings

1. DYNARE Website: http://www.dynare.org/
2. Griffoli, *Dynare User Guide*, 2013.
3. *Dynare Reference Manual*, version 4.5.3, 2017.
4. Canova, *Methods for Applied Macroeconomic Research*, 2007.
5. DeJong and Dave, *Structural Macroeconomics*, 2007.
6. Kydland, Prescott. *The computational experiment: an econometric tool*. Journal of Economic Perspectives, 1996, 10(1):69-85.
7. García-Cicco, Pancrazi, Uribe M. *Real Business Cycles in Emerging Countries*. American Economic Review, 2010, 100(5):2510-2531.

Appendix References and Supplementary Readings

8. Johannes Pfeifer’s collection of Dynare models:

   https://github.com/johannespfeifer/dsge_mod

9. DSGE建模与编程汇总：

   https://mp.weixin.qq.com/s/UrPrtsqXg5kr0hDvr36XgQ